# Written Exam at the Department of Economics winter 2016-17 

## Corporate Finance and Incentives

Final Exam/ Elective Course/ Master’s Course

December 21, 2016
(3-hour closed book exam - access to Excel)

Please note that the language used in your exam paper must correspond to the language for which you registered during exam registration.

## This exam question consists of 3 pages in total

NB: If you fall ill during the actual examination at Peter Bangsvej, you must contact an invigilator in order to be registered as having fallen ill. Then you submit a blank exam paper and leave the examination. When you arrive home, you must contact your GP and submit a medical report to the Faculty of Social Sciences no later than seven (7) days from the date of the exam.

The exam consists of 4 problems. All problems must be solved. The approximate weight in the final grade of each problem is stated. A problem can consist of different sub-questions that do not necessarily have equal weight. Please provide intermediate calculations.

## Problem 1 (CAPM 25\%)

Assume that the risky returns of three stocks have the variance-covariance matrix $A$ and expectation vector $b$ shown in the table below. The risk free interest rate is $r_{f}=1 \%$.

| Stock | Firm 1 | Firm 2 | Firm 3 | Expected return |
| :--- | ---: | ---: | ---: | :---: |
| Firm 1 | 0.38 | -0.23 | 0.13 | $3 \%$ |
| Firm 2 | -0.23 | 0.67 | -0.06 | $9 \%$ |
| Firm 3 | 0.13 | -0.06 | 0.5 | $7 \%$ |

1) Find the minimum-variance portfolio of risky assets. [Solve $A z=\mathbf{1}$ and normalize.]
2) Find the efficient (tangency) portfolio of risky assets. [Solve $A z=b-r_{f} \mathbf{1}$, normalize.]
3) Find expected return and variance for each of the two portfolios from 1) and 2).
4) Suppose you desire an expected return right in the middle between the two expected returns you found in 3). Which portfolio of risky assets achieves this return with least possible variance? What is the variance of this portfolio?

## Problem 2 (Debt and Equity 20\%)

Consider a firm that has been financed by a mix of debt and equity. The debt is a zerocoupon bond that promises bond holders to get 100 million Kroner one year from today.

The firm initially has one short-lived asset which will pay a terminal cash amount one year from today. This amount depends on the state of the world which can be high or low. In the high state, the asset pays 140 million Kroner, in the low state 80 million Kroner.

Capital markets are perfect and no taxes or bankruptcy costs are paid. The risk-free interest rate is $3 \%$. For risk-neutral asset pricing, the high state has probability $40 \%$.

1) Compute the present market values of the debt and equity, and the value of the firm.

The firm has a new, safe investment opportunity. This costs 10 million Kroner today, which can be financed by the equity holders. This investment will deliver 12 million Kroner to the firm next year, in both states. The equity holders may decide on this investment.
2) Compute the present value of next year's 12 million Kroner.
3) Suppose that the equity holders choose to undertake the new investment. What are the new values of debt, equity and the firm?
4) Do you think that the equity holders will choose to invest? Please explain your answer.

## Problem 3 (Option Pricing 30\%)

A non-dividend paying stock currently costs $S_{0}=\$ 20$. In the next two periods its market price $S$ will evolve as shown in the tree below. The risk free rate is $r_{f}=3 \%$ per time period.


1) Compute the risk-neutral probabilities at each node.
2) Consider an American call option with strike price $K=\$ 21$, expiring at time 2 . Compute the market value $C$ of the call option at all nodes in the tree.
3) Consider an American put option with the same strike price $K=\$ 21$, also expiring at time 2. Compute the market value $P$ of the put option at all nodes in the tree.
4) Compute the present value $P V(K)$ of a safe payment of $\$ 21$ at time 2.
5) The put-call parity may be expressed as $S_{0}+P_{0}=P V(K)+C_{0}$ where subscript 0 refers to time 0 . Does this relation hold for these two American options?

## Problem 4 (Various Themes 25\%)

1) Sketch a plot to show how a firm's debt cost of capital and equity cost of capital can be expected to vary with the debt-to-value ratio. Discuss where the weighted average cost of capital (WACC) curve may lie, when also seen as a function of the debt-to-value ratio.
2) The random returns of assets over a period of time may be modeled as random variables $r_{i}(j)$ that can take on $m$ possible values. Suppose there exists one vector $p$ such that every asset $i$ satisfies $0=p_{1} r_{i}(1)+\ldots+p_{m} r_{i}(m)$. Does this rule out arbitrage opportunities? Do you need to know more about the vector $p$ ? What could be the interpretation of vector $p$ ?
3) The covered interest parity for the dollar-euro exchange rate claims that

$$
F=S \frac{1+r_{\S}}{1+r_{€}}
$$

Explain what $F$ and $S$ would mean in this equation. Briefly explain the derivation of this.

